ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM OPTIMIZATION USING PSO FOR PREDICTING SEDIMENT TRANSPORT IN SEWERS

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ABSTRACT

The flow in sewers is a complete three phase flow (air, water and sediment). The mechanism of sediment transport in sewers is very important. In other words, the passing flow must be able to wash deposited sediments and the design should be done in an economic and optimized way. In this study, the sediment transport process in sewers is simulated using a hybrid model. In other words, using the Adaptive Neuro-Fuzzy Inference System (ANFIS) and the Particle Swarm Optimization (PSO) algorithm a hybrid algorithm (ANFIS-PSO) is developed for predicting the Froude number of three phase flows. This inference system is a set of if-then rules which is able to approximate non-linear functions. In this model, PSO is employed for increasing the ANFIS efficiency by adjusting membership functions as well as minimizing error values. In fact, the PSO algorithm is considered as an evolutionary computational method for optimizing the process continues and discontinues decision making functions. Additionally, PSO is considered as a population-based search method where each potential solution, known as a swarm, represents a particle of a population. In this approach, the particle position is changed continuously in a multidimensional search space, until reaching the optimal response and or computational limitations. At first, 127 ANFIS-PSO models are defined using parameters affecting the Froude number. Then, by analyzing the ANFIS-PSO model results, the superior model is presented. For the superior model, the Mean Absolute Percentage Error (MAPE), Root Mean Square Error (RMSE) and the determination coefficient ($R^2$) were calculated equal to 5.929, 0.324 and 0.975, respectively.

Keywords: Sediments; Circular channel; Hybrid model; ANFIS; Particle Swarm Optimization

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1. INTRODUCTION

Due to the importance of this topic and the mechanism of sediment transport in sewers, there are many experimental, analytical and numerical studies carried out on it. Novak and Nalluri [1] conducted a study to investigate the critical threshold speed and concluded that there is a good fitness because of using the dimensionless parameter $D_h/y$ for demonstrating the bed shape influence. They also concluded that the required slope for sewers smaller than 500mm is steeper than bigger sewers. Macke [2] conducted three sedimentation experiments using three smooth pipes (diameters of 192mm, 290mm and 445mm) with full flow by means of sands with average diameters varying 0.16mm to 0.37mm moving as suspended loads. He provided the results of the experiment as a relationship. In addition, May et al. [3] carried out a number of experiments with shallow sedimentation depths in a pipe in which sediment transport was taking place in the form of bed loads. The results obtained from their study indicated that in the presence of limited sedimentation depths, more sediment transport capacity is expected. Nalluri and Kithsiri [4] proposed a formula for calculating the friction coefficient $(\lambda_s)$ using the combination of results of other studies conducted on rectangular channels with rough beds for transporting non-cohesive sediments as bed load in without sedimentation conditions and included wider domain of data. Later, Ota and Nalluri [5] by conducted an analytical study to examine the sediment transport occurring in circular channels. By analyzing the results of the study, they showed that sediments with bigger dimensions require greater discharge for transporting. They calculated sediment concentration changes in terms of ppm versus parameters such as channel sediments, channel diameter and standard deviation. Sutter et al. [6] studies the relationships provided by other researchers in the field of the sediment transport criterion in sedimentation limit conditions and indicated that these relationships have a significant error in predicting initial erosion and rate of volume fractional for non-uniform particles of sewage sediments. They also stated that these relationships yield better results for finer sediments than graded materials. They evaluated variations of bed shear stress versus bed load and demonstrated that by increasing bed load the amount of bed stress increases. In recent decades, various neural network and artificial intelligence techniques have been used by many researchers in predicting and pattern cognition of complex, difficult and non-linear phenomena of different sciences. In addition, recently neuro-fuzzy models and various neural network algorithms have been applied for solving various problems of hydrology and hydraulics. Suspended sediment load was predicted by Kisi [7]. Using ANN and Neuro fuzzy models, Kisi modeled the sediment growth curve and the multiple linear regressions of observational values. Azamathulla et al. [8] using the Gene Expression Programming (GEP) and ANFIS, studied suspended sediment in natural rivers. By analyzing the results of their own study, they showed that the GEP model predicts observational values with higher accuracy. Subsequently, Rajaee [9] modeled values of suspended sediment load using the wavelet and Neuro-fuzzy models. They showed that the introduced models are very efficient in modeling of sediments. Most of the relationships provided for predicting sediment transport by means of the self-washing concept are regression relationships. The basic issue of these methods is...
the lack of good performance in different situations than what is used for estimating the relationship. In this study, due to the good performance of based on artificial intelligence methods in various engineering sciences, the minimum velocity required for preventing from sedimentation is predicted using a new hybrid method based on the ANFIS and PSO (ANFIS-PSO). To this end, the parameters affecting the Froude number are identified and by conducting a sensitivity analysis all possible combinations are determined and eventually the superior model is introduced.

2. METHODS AND MATERIALS

2.1 Adaptive neuro-fuzzy inference system (ANFIS)

ANFIS is a fuzzy inference model in the framework of multi-layer fuzzy networks developed by Jang [10]. The structure of fuzzy networks is created in two conditions including Mamdani and Sugeno. If conditional rules constructing the fuzzy network are as fuzzy sentences in antecedent and consequent, the Mamdani network is created. If this rule is as a function of inputs or a constant function, the Sugeno-fuzzy network is formed. In the situation where consequent parameters are fixed in the Sugeno network, the Sugeno-fuzzy network is zero degree and if they are as a function of \( n \) degree, the Sugeno network. Neuro-fuzzy systems have a more flexible structure than other artificial intelligence networks and are able to create connection between input and output in non-linear complex systems in a simpler way. These systems employ the learning ability of the neural network for correcting membership function coefficients which are most important members of fuzzy inference systems.

\[
R_i : \text{IF} \ x = A_i \ \text{and} \ y = B_i \ \text{THEN} \ f_i = p_i x + q_i y + r_i \quad (1)
\]

\[
R_j : \text{IF} \ x = A_j \ \text{and} \ y = B_j \ \text{THEN} \ f_j = p_j x + q_j y + r_j \quad (2)
\]

In these equations, \( A_1, A_2, B_1, B_2 \) are membership functions of the inputs \( x \) and \( y \) and the coefficients \( \{ p_1, p_2, q_1, q_2, r_1, r_2 \} \) are output function parameters. Five different layers of ANFIS are defined as follows:

First layer: each node in this layer represents the membership degree of input parameters.

\[
O_{1,i} = \mu A_i(x) \quad i = 1,2
\]

\[
O_{1,3} = \mu B_i(x) \quad i = 3,4
\]

In these equations, \( x \) and \( y \) are inputs of the node \( i \) and \( A_i \) and \( B_i \) are the fuzzy set related to this node. Also, \( O_{1,i} \) represents the membership degree of the fuzzy set. Membership functions can be in different forms, but due to the good performance of the Guassian membership function in different hydraulic problems, this function is used as follows:

\[
\mu_A(x) = \exp \left( \frac{(x_i - c_i)^2}{2\sigma_i^2} \right) \quad (5)
\]
where, $\sigma_i$ and $c_i$ are the width and the center of the Gaussian membership function known as premise parameters.

Second layer: each node in this layer is denoted by the label $P$. In this layer, each node calculates the activity degree of a rule.

$$O_{i,2} = w_i = \mu_A(x) \times \mu_B(y) \quad i = 1,2$$ (6)

where, $\mu_A$ and $\mu_B$ are the membership functions $x$ and $y$, respectively.

Third layer: Nodes of this layer are denoted by the label $N$. In this layer, the ratio of the activity degree of the rule $i^{th}$ to the sum of all rule activity degrees (firing strength) is calculated as follows:

$$O_{i,3} = \bar{w}_i = w_i / (w_1 + w_2) \quad i = 1,2$$ (7)

Fourth layer: each node in this layer is as a square node which its output is computed as the following function:

$$O_{i,4} = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i)$$ (8)

Fifth layer: Each node in this layer is as a circular node. This layer presents the sum of input signals as the total output as follows:

$$O_{i,5} = \sum \bar{w}_i f_i = \frac{\sum w_i f_i}{\sum \bar{w}_i}$$ (9)

2.2 Particle swarm optimization (PSO)

The PSO is a meta-heuristic approach introduced by Kennedy and Eberhart [11] inspired from the social behavior of birds. The PSO provides the possibility of local and global search and fast convergence to global optimization by adjusting simple parameters. The main advantage of this method over other minimization strategies is that abundant amount of swarming particles makes this method stable against the local minimizing problem (Shi and Eberhart [12], EL-Zonkoly [13]). In this algorithm, first, an initial response set is created so that each particle is assumed as a possible solution. Then, for finding the optimized response in the possible response space, the response search is conducted by applying generations. Each particle is defined as two-dimensional with two situation and velocity values and in each step of particle movement the best response in terms of fitness is determined for all particles. In other words, the optimized obtained from each step and the optimized obtained from all steps are known as pbest and gbest, respectively. Furthermore, all particles obtained based on pbest and gbest, update their situations to achieve the global optimized solution. If the coordinates $x$ and $y$ related to a $N$ member set of particles are considered as $x_i$ and $y_i$ and its velocity along the axes $x$ and $y$ as $v_{x,i}$ and $v_{y,i}$, velocity and situation of each particle in each iteration are updated by following relationships:
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\[ \begin{aligned}
  v_x(t+1) &= w(t)v_x(t) + c_1 \cdot \text{rand}(pbest_x, x(t)) + c_2 \cdot \text{rand}(gbest_x, x(t)) \\
  v_y(t+1) &= w(t)v_y(t) + c_1 \cdot \text{rand}(pbest_y, y(t)) + c_2 \cdot \text{rand}(gbest_y, y(t)) \\
  x_i(t+1) &= x_i(t) + v_x(t+1) \\
  y_i(t+1) &= y_i(t) + v_y(t+1)
\end{aligned} \]  

(10)

where, \( t=1,2,\ldots,I_{\max} \), \( i=1,2,\ldots,N \) represents the number of iteration, \( I_{\max} \) is the maximum iteration, \( \text{rand()} \) produces a random value on the domain \([0,1)\), \( c_1 \) and \( c_2 \) are two positive constant values entitled "cognition learning rate" and "social learning rate, \( pbest_x(t) \) and \( pbest_y(t) \) are the best responses obtained in the \( t \)th iteration in the direction of the axes \( x \) and \( y \) and \( w(t) \) is inertia weight calculated by the following relationship:

\[ w(t) = w_{\max} \frac{(w_{\max} - w_{\min}) \times I}{I_{\max}} \]  

(12)

where, \( w_{\min} \) and \( w_{\max} \) are the minimum and maximum inertia weights, respectively.

2.3 ANFIS-PSO hybrid method

In this study, the PSO approach is employed for determining the ANFIS membership functions. The main advantage of this method is reducing computational costs for a given topology. In this section, the algorithm presented in this study for providing the ANFIS-PSO hybrid method is described step by step as follows:

At first, using a matrix comprising data related to sediment transport including 2 to 7 input columns and one column as the output or the objective, data are arranged in order to estimate the minimum velocity by ANFIS. Using arranged data, the ANFIS training is started. The training procedure allows this system to adjust defined parameters as the model input or output. The training process is terminated once criteria defined for stopping the program such as reaching to the minimum specified for the objective function or finishing the number of specified iterations are satisfied. After determination of training data, the type of membership functions and the fuzzy inference system are optimized by adapting the membership function parameters. In this study, PSO is used for determination of parameters depending on membership functions in the fuzzy inference system. Then, a vector with \( N \) dimensions, where \( N \) is the number of membership functions is defined. This vector includes membership function parameters which their values are optimized by PSO. The fitness function defined in this study is as the Mean Square Error (MSE) function. Next, parameters related to the PSO algorithm are defined which their values obtained by trial and error as listed in Table 1. At first, parameters are randomly determined, then determined values are updated using the PSO algorithm. In each step of iteration, one of the membership function parameters is updated. For example, in the first iteration the parameter \( c_i \) and in the next iteration, \( \sigma_i \) is updated and this procedure continues for all parameters after updating of all parameters once, the first parameter is updated for the second time in the next iteration. Therefore, all parameters are updated as iteration by iteration in order to reaching to the optimal point. The application of PSO in order to optimize the membership function parameters includes: 1) In this step a situation and initial velocity are determined with the
same size for all particles. 2) Determination of the pbest and gbest values. If the obtained value is better than the current value of the particle situation, pbest is updated and its value is replaced by new value. Furthermore, if the best value of particle single values is better than the gbest total value, the situation of best particles is updated as well. 3) Examination of each particle fitness (pbest) and saving of particles with the best fitness value (gbest). 4) Correction of velocities in terms of updated values of gbest and pbest. 5) Updating of particles. 6) This process continues until finishing the number of iterations. Now, ANFIS outputs which their parameters have been obtained by PSO are extracted. Finally, outputs related to the prediction of the ANFIS-PSO hybrid method are determined.

2.4 Experimental model

In this study, the experimental measurements obtained by Ab. Ghani [14], Ota et al. [5] and Vongvisessomjai et al. [15] are employed for validating the numerical model. In Table 2, the range of the experimental data used in this paper is shown. In this Table, Fr is the flow Froude number, Cv is the volumetric concentration of particles, d/R is the ratio of the average diameter of sediment particles to the hydraulic radius, D^2/A is the ratio of the pipe (channel) diameter to the flow cross-section, R/D is the ratio of the hydraulic radius to the pipe diameter, D_{gr} is the dimensionless parameter number \((d_{50}(g(s-1)/\nu^2)^{1/3})\), d/D is the ratio of the average diameter of particles to the pipe diameter and \(\lambda_s\) is the general resistance coefficient of sediment load (sedimentary flow resistance coefficient).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>minimum</th>
<th>maximum</th>
<th>mean</th>
<th>variance</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>discharge ((L/s))</td>
<td>4.678E-01</td>
<td>9.522E+01</td>
<td>1.809E+01</td>
<td>2.710E+02</td>
<td>1.646E+01</td>
</tr>
<tr>
<td>Pipe diameter ((mm))</td>
<td>1.000E+02</td>
<td>4.500E+02</td>
<td>2.748E+02</td>
<td>7.955E+03</td>
<td>8.919E+01</td>
</tr>
<tr>
<td>Flow depth ((m))</td>
<td>2.000E-02</td>
<td>2.291E-01</td>
<td>1.103E-01</td>
<td>2.980E-03</td>
<td>5.459E-02</td>
</tr>
<tr>
<td>Hydraulic radius ((m))</td>
<td>1.200E-02</td>
<td>1.138E-01</td>
<td>5.725E-02</td>
<td>6.326E-04</td>
<td>2.515E-02</td>
</tr>
<tr>
<td>Cross-section ((m^2))</td>
<td>1.401E-03</td>
<td>8.136E-02</td>
<td>2.523E-02</td>
<td>4.147E-04</td>
<td>2.036E-02</td>
</tr>
<tr>
<td>velocity ((mls))</td>
<td>2.370E-01</td>
<td>1.216E+00</td>
<td>6.675E-01</td>
<td>3.902E-02</td>
<td>1.975E-01</td>
</tr>
<tr>
<td>Mean diameter of particles ((mm))</td>
<td>2.000E-01</td>
<td>8.300E+00</td>
<td>3.035E+00</td>
<td>6.209E+00</td>
<td>2.492E+00</td>
</tr>
<tr>
<td>Channel slope</td>
<td>5.070E-04</td>
<td>6.000E-03</td>
<td>3.017E-03</td>
<td>2.564E-06</td>
<td>1.601E-03</td>
</tr>
<tr>
<td>general resistance of sediment load</td>
<td>1.290E-02</td>
<td>5.320E-02</td>
<td>2.697E-02</td>
<td>8.826E-05</td>
<td>9.395E-03</td>
</tr>
</tbody>
</table>
2.5 Combinations of ANFIS-PSO models

In this section, the combinations of the numerical model input parameters are evaluated. In other words, seven input parameters including $C_v$, $d/R$, $D^2/A$, $R/D$, $D_{gr}$, $d/D$ and with different combinations are used for modeling the parameter $F_r$. It should be noted that 127 different models are developed by combining the introduced parameters. The combinations of the mentioned parameters are shown in Fig. 1.

![Figure 1. Combination of ANFIS-PSO models](image_url)

2.6 Criteria for studying numerical model accuracy

In the current study, the root mean square error (RMSE), the mean absolute percent error (MAPE), the scatter index ($SI$) and the determination coefficient ($R^2$) are used as follows:
where \( R_{\text{observed}} \), \( R_{\text{predicted}} \), \( \bar{R}_{\text{observed}} \) and \( n \) are experimental values, results predicted by the numerical model, the average of experimental values and the number of experimental measurements.

3. RESULTS AND DISCUSSION

3.1 Superior models

In the following, the results of the numerical models are investigated. Seven models with one input parameter predict values of the flow Froude number. Among all models with one input parameter, model 6 estimates experimental values with higher accuracy in both training and test cases. The values of \( R^2 \) for Model 6 in both training and test modes are calculated 0.705 and 0.724, respectively. Furthermore, for this model, the values of SI and RMSE in the test mode are 0.271 and 1.068, respectively. It should be noted that this model is a function of the ratio of the average diameter of particles to the pipe diameter \( (d/D) \). In addition, 21 models are produced by combining two input parameters (model8 to model28). Among all models simulating values of the Froude number with a combination of two input parameters, model 8 has the highest accuracy. This model estimates values of the Froude number in terms of the volumetric concentration of sediments and the ratio of the average diameter of sediment particles to the hydraulic radius. The values of \( \text{RMSE}, \text{MAPE} \) and SI for this model in the training mode are obtained 0.289, 5.542 and 0.070, respectively. Furthermore, the values of \( \text{RMSE} \) and \( R^2 \) for the mentioned mode are calculated 0.324 and 0.975, respectively. In the following, the results of the models with three input parameters are evaluated. In this case, 35 different models are defined for predicting the Froude number. In other words, models 29 to 63 mode values of the Froude number with a combination of three input parameters. Based on the analysis of the numerical model results, model 33 has the highest correlation and the lowest accuracy among the models with three input parameter. This model simulates objective function values in terms of \( C_v, d/R \) and \( \lambda_v \). In both training and testing modes, the \( \text{RMSE} \) value is 0.340 and 0.329, respectively. In addition, 35 different models with a combination of four input parameters are defined. Models 64 to 98 are produced with a combination of four input parameters. Among the
mentioned models, model 69 has the highest accuracy. Based on the modeling results, this model simulates Froude number values with reasonable accuracy. For example, the values of RMSE and SI for the training mode of model 69 are calculated 0.377 and 0.092, respectively. In addition, according to the combinations of the input parameters, model 69 estimates values of the Froude number in terms of $C_v$, $d/R$, $R/D$, and $d/D$. It is concluded that 21 distinctive models with a combination of five input parameters are defined for modeling the Froude number of a three-phase flow. In other words, models 99 to 119 are produced by four input parameters. According to the modeling results, among models 99 to 119, model 109 has the highest accuracy. For this model, in the training mode, the values of RMSE, MAPE, and $R^2$ are approximately 0.425, 9.773, and 0.951, respectively. Furthermore, the SI value in the training mode for this model is almost equal to 0.103. In addition, the values of MAPE, SI, and $R^2$ of this model in the test mode are calculated 11.231, 0.142, and 0.962, respectively. In the next section, the accuracy of the ANFIS-PSO model produced by combining six input parameters is evaluated. In this regard, seven different models are developed by combining six input parameters. In other words, models 120 to 126 predict Froude number values with a combination of six input parameters. By analyzing the mentioned model results, the most accurate model is model 123. For this model, the $R^2$ values for both training and testing modes are obtained 0.924 and 0.902, respectively. In contrast, the values of SI and R2 in the test mode of model 123 are calculated 0.162 and 0.902, respectively. In the next section, a model estimating Froude number values of the three-phase flow with a combination of seven input parameters is evaluated. In other words, this model simulates Froude number values in terms of the volumetric concentration of particles, the ratio of the average diameter of sediment particles to the hydraulic radius, the ratio of the pipe (channel) diameter to the flow cross-section, the ratio of the hydraulic radius to the pipe diameter, the dimensionless particle number, the ratio of the average diameter of particles to the pipe diameter and the general resistance coefficient of sediment load. According to the modeling results, the SI value in the training and testing modes of model 127 are calculated 0.121 and 0.154, respectively. Nonetheless, the values of MAPE and $R^2$ for the test mode of model 127 are calculated 13.863 and 0.911, respectively. Next, the error distribution of the superior models including models 6, 8, 33, 69, 109, 123 and 127 which have more accuracy among the models with 1, 2, 3, 4, 5, 6 and 7 input parameters is shown. Furthermore, the results of the error distribution of the mentioned models in the training and testing modes are shown. For example, in the training mode of ANFIS-PSO models, model 6 predicts about 50% of the numerical results with an error equal to 16%. Nonetheless, model 8 estimates about 91% of the Froude number values with an error less than 12%, while model 33 predicts about 73% of its results with an error less than 8%. According to the modeling results, in the training mode of model 69, predicts about 69% of the Froude number values with an error less than 8%. According to the modeling results, in the training mode of model 69, the Froude number values have allocated an error less than 10% to themselves. For model 109, about 38% of the Froude number values have an error less than 6%. Furthermore, in the training mode of model 127 predicts 82% of the numerical results with an error less than 18%. In the following, the error distribution of the test mode of the numerical models is studied. For example, for model 6, the results of 29% of the numerical models have an error less than 8%. However, for model 8, about 88% of the Froude number results simulated by this model have an error less than 12%. In addition,
about 94% of the results simulated by model 33 have an error less than 14%. For model 69, about 23% of the results estimated by the mentioned model have an error less than 4%. In other words, according to the error distribution results, for model 109 about 56% of the simulated values have an error less than 10%. Also, for model 123, about 72% of the simulated values of the Froude number have an error less than 16%. Furthermore, 71% of the numerical values simulated by model 127 have an error less than 18%.

Figure 2. Graphs of superior models error distribution in (a) training mode (b) testing mode

According to the analysis of the results of 127 ANFIS-PSO models, model 8 is introduced as the superior model. This model predicts Froude number values of the three-phase flow in terms of \( C_v \) (volumetric concentration of particles) and \( d/R \) (the ratio of the average diameter of sediment particles to the hydraulic radius). In Table 8, the results of the statistical indices for model 8 are listed. In addition, in Fig. 3, the comparison of the predicted Froude number values by model 8 is shown in both training and test modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>( R^2 )</th>
<th>MAPE</th>
<th>RMSE</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train</td>
<td>0.978</td>
<td>5.542</td>
<td>0.289</td>
<td>0.070</td>
</tr>
<tr>
<td>Test</td>
<td>0.975</td>
<td>5.929</td>
<td>0.324</td>
<td>0.082</td>
</tr>
</tbody>
</table>
The sensitivity of model 8 was examined based on the discrepancy ratio (DR) parameter. Discrepancy ratio is the ratio of predicted to experimental results as follows:

\[ DR = \frac{R_{(Predicted)}}{R_{(Observed)}} \]  

(17)

The discrepancy ratio changes versus Froude number values are shown for the superior model in Fig. 4. An average discrepancy ratio (\(DR_{(ave)}\)) close to 1 represents the proximity between predicted and experimental values. For Model 8, the \(DR_{(ave)}\) values for training and testing are 0.944 and 1.015, respectively.

4. CONCLUSIONS

In general, circular channels are widely used in urban sewage disposal systems. Furthermore, the flow within this type of channels is a three-phase flow composed of water, air and sediments. However, many studies have been conducted on the flow within sewage channels by different researchers. In this study, a hybrid approach was introduced for studying the three-phase flow characteristics. In the following, the most important results of
the current study are provided. For models with two input parameters, the model estimated the Froude number in terms of $C_v$ and $d/R$. For this model in the training mode, the values of RMSE, MAPE and SI were calculated 0.289, 5.542 and 0.070, respectively. This model predicted the objective function values in terms of $C_v$, $d/R$ and $\lambda_i$. The RMSE value for this model in the training mode was equal to 0.340.

REFERENCES