Modified Sine-Cosine Algorithm for Sizing Optimization of Truss Structures with Discrete Design Variables

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ABSTRACT

This paper proposes a modified sine cosine algorithm (MSCA) for discrete sizing optimization of truss structures. The original sine cosine algorithm (SCA) is a population-based metaheuristic that fluctuates the search agents about the best solution based on sine and cosine functions. The efficiency of the original SCA in solving standard optimization problems of well-known mathematical functions has been demonstrated in literature. However, its performance in tackling the discrete optimization problems of truss structures is not competitive compared with the existing metaheuristic algorithms. In the framework of the proposed MSCA, a number of worst solutions of the current population is replaced by some variants of the global best solution found so far. Moreover, an efficient mutation operator is added to the algorithm that reduces the probability of getting stuck in local optima. The efficiency of the proposed MSCA is illustrated through multiple benchmark optimization problems of truss structures.

Keywords: discrete optimization; sizing optimization; truss structures; metaheuristic; sine cosine algorithm.

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1. INTRODUCTION

The material cost is an important factor in the construction of structures and from an economical viewpoint, the design of a minimum weight is the best structure. In order to find such designs, structural optimization techniques can be effectively used. In the last decade, many optimization techniques have been developed and successfully applied to a wide range of structural optimization problems including sizing, layout and topology optimization problems [1-3]. Metaheuristics are the most general kinds of stochastic optimization algorithms and they are now recognized as one of the most practical approaches for solving...
a wide range of optimization problems. The main idea behind designing these metaheuristic algorithms is to solve complex optimization problems where other optimization methods have failed to be effective. The practical advantage of metaheuristics lies in both their effectiveness and general applicability. In recent years, metaheuristic algorithms are emerged as the global search approaches which are responsible to tackle the complex optimization problems.

Most of the metaheuristic algorithms are developed based on natural phenomena. Every metaheuristic method consists of a group of search agents that explore the design space based on randomization and some specified rules inspired the laws of natural phenomena. For example, Genetic Algorithms (GA) [4], Biogeography-Based Optimization (BBO) [5], and Differential Evolution (DE) [6] are developed based on the Darwin’s principle of survival of the fittest. Gravitational Search Algorithm (GSA) [7], Colliding Bodies Optimization (CBO) [8] and Center of Mass Optimization (CMO) [9] are Physics-based metaheuristic algorithms. Particle Swarm Optimization [10] (PSO), Ant Colony Optimization [11] (ACO), Bat algorithm [12] (BA) and Dolphin Echolocation Algorithm (DEA) [13] are recognized as popular Swarm intelligence metaheuristics. One of the newly developed metaheuristic algorithms is the Sine Cosine Algorithm (SCA), which is proposed by Mirjalili [14]. The SCA requires that the generated solutions fluctuate outwards or towards the best solution found so far using sine and cosine functions. It was demonstrated in [14] that the SCA is able to effectively solve the continuous optimization problems.

Optimization of truss structures is very popular in the area of structural optimization and over the last decades, various algorithms have been proposed for solving these problems. There is a significant number of metaheuristics employed for truss optimization with discrete variables in the literature such as: Discrete Heuristic Particle Swarm Ant Colony Optimization (DHPSACO) [15], Improved Dolphin Echolocation Algorithm (IDEA) [16], Improved Mine Blast Algorithm (IMBA) [17], Adaptive Elitist Differential Evolution (AEDE) [18], and Improved Fireworks Algorithm (IFWA) [19]. In the present study, SCA is focused and a modified Sine Cosine algorithm (MSCA) is proposed to handle the truss structures optimization with discrete design variables. In the MSCA two main strategies are followed for the exploration and exploitation of the design space. In the first strategy, some of worst solutions in each iteration are removed and the same number of variants of the best solution is added to the population. In the second strategy, a mutation operator is added to the algorithm. Five benchmark optimization problems of truss structures with discrete variables are presented and the results of MSCA are compared with literature.

2. TRUSS OPTIMIZATION PROBLEM

For the optimization problem of trusses, objective function is the structural weight and some limitations are usually considered on nodal displacements and element stress as the design constraints. Formulation of truss structures optimization problem is as follows:

Minimize: \[ W = \sum_{i=1}^{n} \gamma_i l_i x_i, \quad i = 1, 2, \ldots, n \]
Subject to:

\[
\begin{align*}
g_j' &= \frac{d_j}{\bar{d}_j} - 1, \quad j = 1, 2, \ldots, m \\
g_k' &= \frac{\sigma_k}{\bar{\sigma}_k} - 1, \quad k = 1, 2, \ldots, n
\end{align*}
\]

(2)

Subject to:

\[X_i^L \leq X_i \leq X_i^U\]

(3)

where \(W\) is structural weight; \(\gamma_i\), \(l_i\) and \(X_i\) are the density of material, element length and cross-sectional area of \(i\)th element, respectively; displacement and stress constraints are represented by \(g_j^d\) and \(g_k^s\), respectively; \(d_j\) and \(\sigma_k\) are \(j\)th node displacement and \(k\)th element stress, respectively; \(\bar{d}_j\) and \(\bar{\sigma}_k\) are their allowable values; \(n\) and \(m\) are numbers of elements and nodes, respectively.

The following exterior penalty function (EPF) is employed to handle the constraints of the above constrained optimization problem.

\[
\phi = W \times \left(1 + r_p \sum_{j=1}^{n} \max(0, g_j^d)^2 + r_p \sum_{i=1}^{n} \max(0, g_k^s)^2\right)
\]

(4)

where \(\phi\) is pseudo unconstrained objective function; and \(r_p\) is a penalty parameter. In this study, \(r_p\) is linearly increased from 1.0 at the first iteration to \(10^6\) at the last one during the optimization process.

3. SINE COSINE ALGORITHM

In essence, all population-based metaheuristic algorithms explore the design space using a number of search agents which follow a set of updating rules. These updating rules play an important role in performance of the metaheuristic algorithms. In the Sine Cosine Algorithm (SCA) [14] the following equation is used as the updating rule of position of population in the design space:

\[
X'_{i,j} = \begin{cases} 
X_{i,j} + r_t \times \sin(r_t) \times \left[f_j \times P_j' - X_{i,j}\right], & r_t < 0.5 \\
X_{i,j} + r_t \times \cos(r_t) \times \left[f_j \times P_j' - X_{i,j}\right], & r_t \geq 0.5
\end{cases}
\]

(5)

\[
r_t = a \times \left(1 - \frac{t}{t_{\text{max}}}\right)
\]

(6)

where \(X_{i,j}'\) and \(X'_{i,j}\) are the \(j\)th design variable of the \(i\)th solution at iterations \(t+1\) and \(t\), respectively; \(\sin(.)\) and \(\cos(.)\) represent the sine and cosine mathematical functions; \(P_j'\) is the \(j\)th design variable of the best solution; \(a\) is a constant and in this study \(a = 2.0\); \(t_{\text{max}}\) is the
maximum number of iterations; \( r_2 \) and \( r_4 \) are random numbers in \([0, 2\pi]\) and \([0,1]\), respectively; and \( r_3 \) is a random number that \( r_3 > 1 \) and \( r_3 < 1 \) emphasize and de-emphasize the effect of the best solution in defining the distance.

It was demonstrated in [14] that the original SCA has proper performance in solving standard optimization problems of well-known mathematical functions. The computational experience of the present study however reveals that the SCA is not an efficient metaheuristic algorithm for discrete sizing optimization of truss structures.

### 4. MODIFIED SINE COSINE ALGORITHM

In order to improve the performance of the SCA in dealing with the discrete sizing optimization problems of truss structures two computational strategies are implemented and the improved metaheuristic is named as modified Sine Cosine algorithm (MSCA). The proposed strategies, termed here as Regeneration and Mutation, are described below. In addition, updating rule of position of population in the discrete MSCA is as follows:

\[
X_{i,j}^{t+1} = \begin{cases} 
\text{round} \left( X_{i,j}^t + r_1 \times \sin(r_2) \times \left| r_3 \times P_j^t - X_{i,j}^t \right| \right), & r_4 < 0.5 \\
\text{round} \left( X_{i,j}^t + r_1 \times \cos(r_2) \times \left| r_3 \times P_j^t - X_{i,j}^t \right| \right), & r_4 \geq 0.5
\end{cases}
\]  

(7)

where \( \text{round}(.) \) rounds numbers to their nearest integer.

**Regeneration:** in each iteration of the optimization process, the population, including \( np \) particles, is sorted in an ascending order based on the objective function values of particles as represented below:

\[
\text{sort}(X^t) = [X_1^t X_2^t ... X_{j-1}^t X_j^t X_{j+1}^t ... X_{np-1}^t X_{np}^t]
\]

(8)

where \( \text{sort}(X^t) \) is the sorted current population; and \( X_j^t \) to \( X_{np}^t \) are the worst solutions at iteration \( t \) that should be regenerated.

Then, a number of \( \lambda \times np \) worst particles (i.e. \( X_1^t \) to \( X_{np}^t \)) are removed from the population and instead, the best solution found so far, \( X^* = [X_1^* X_2^* ... X_j^* ... X_n^*]^{\top} \), is copied \( \lambda \times np \) times in the population. In these solutions, except the last one, one randomly selected design variable is regenerated in the design space on a random basis as follows:

\[
X_j^t \rightarrow [X_1^* X_2^* ... X_{j-1}^* X_j^* X_{j+1}^* ... X_{np-1}^* X_{np}^t], \; l = k,...,np-1
\]

(9)

in which

\[
X_j^* = \text{round} \left( X_j^t + r \times (X_j^u - X_j^l) \right), \; j \in [1,2,...,n]
\]

(10)

where \( X_j^l \) and \( X_j^u \) are lower and upper bounds of the \( j \)th design variable; and \( r \) is a random
number in $[0,1]$. The regenerated design variables of particles $X_k'$ to $X_{np}'$ are substituted in the last particle ($X_{np}'$). This strategy will increase the probability of finding the promising regions of the design space.

**Mutation:** In the framework of MSCA, a mutation operation is implemented to reduce the probability of trapping into local optima. In this way, a mutation rate of $mr$ is considered and for each particle ($X_i, i=1,2,...,np$) a random number in $[0,1]$ is selected in each iteration. If for the $i$th particle, the selected random number is less than $mr$, $X_i$ will be regenerated in the design space as follows:

$$X_i^{t+1} = \text{round} \left( X_i^t + \frac{1}{t_{\text{max}}} \times R^i \otimes (X_{\text{best}}^t - X_i^t) \right)$$

(11)

where in iteration $t$, $R^i$ is a vector of random numbers in $[0,1]$; $X_{\text{best}}^t$ is the best particle of the current population; and $X_i^t$ is a randomly selected particle from the current population.

In the framework of MSCA, a simple mechanism is employed to return into the feasible region the agents that violate side constraints. During the optimization process, if a design variable violates the side constraints, it will be replaced by the lower/upper bound as follows:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^L & \text{if } X_{i,j}^{t+1} < X_{i,j}^L \\ X_{i,j}^U & \text{if } X_{i,j}^{t+1} > X_{i,j}^U \end{cases}$$

(12)

where $X_{i,j}^L$ and $X_{i,j}^U$ are respectively the lower and upper bounds of the $j$th design variable of the $i$th solution.

The best combination of internal parameters $\lambda$ and $mr$ is determined by performing sensitivity analysis. In this way, $\lambda \in \{0.1, 0.2, 0.3\}$ and $mr \in \{0.01, 0.05, 0.10\}$ are considered and for each combination of these two parameters, 20 independent optimization runs are conducted. The results of this study demonstrate that the best combination is $\lambda=0.2$ and $mr=0.05$.

The flowchart of MSCA is depicted in Fig. 1.

5. NUMERICAL RESULTS

In order to illustrate the merit of the proposed MSCA, a number of popular discrete benchmark truss optimization problems are presented and the obtained results are compared with those of literature. For the presented examples, 20 independent optimization runs are performed and the best weight (Best), average weight (Average) and the standard deviation (SD) of optimal weights are reported.
5.1 Example 1: 10-bar planar truss

The 10-bar truss shown in Fig. 2 is one of the most extensively studied problems. The vertical load in nodes 2 and 4 is equal to $10^5$ lb. The Young's modulus and density of material are $10^4$ ksi and 0.1 lb/in$^3$, respectively.

The allowable stress for all members is specified as 25 ksi both in tension and
compression. The maximum displacements of all free nodes in the \(x\) and \(y\) directions are limited to \(\pm 2\) in. In this example, the discrete design variables are selected from the following list: [1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50] (in²).

Cross-sectional areas of elements 1 to 10 (i.e. \(A_1\) to \(A_{10}\)) are considered as the design variables. In the optimization process, 50 particles are involved and the maximum number of iterations is chosen to be 200.

The optimization results of SCA and MSCA are compared with those of HPSO [20], HHS [21], and AEDE [18] in Table 1. In addition, the convergence curves of SCA and MSCA are compared in Fig. 3.

<table>
<thead>
<tr>
<th>Design variables</th>
<th>HPSO</th>
<th>HHS</th>
<th>AEDE</th>
<th>SCA</th>
<th>MSCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td>30.0</td>
<td>33.5</td>
<td>33.5</td>
<td>26.5</td>
<td>33.5</td>
</tr>
<tr>
<td>(A_2)</td>
<td>1.62</td>
<td>1.62</td>
<td>1.62</td>
<td>2.62</td>
<td>1.62</td>
</tr>
<tr>
<td>(A_3)</td>
<td>22.9</td>
<td>22.9</td>
<td>22.9</td>
<td>26.5</td>
<td>22.9</td>
</tr>
<tr>
<td>(A_4)</td>
<td>13.5</td>
<td>14.2</td>
<td>14.2</td>
<td>18.8</td>
<td>14.2</td>
</tr>
<tr>
<td>(A_5)</td>
<td>1.62</td>
<td>1.62</td>
<td>1.62</td>
<td>1.62</td>
<td>1.62</td>
</tr>
<tr>
<td>(A_6)</td>
<td>1.62</td>
<td>1.62</td>
<td>1.62</td>
<td>2.38</td>
<td>1.62</td>
</tr>
<tr>
<td>(A_7)</td>
<td>7.97</td>
<td>7.97</td>
<td>7.97</td>
<td>11.5</td>
<td>7.97</td>
</tr>
<tr>
<td>(A_8)</td>
<td>26.5</td>
<td>22.9</td>
<td>22.9</td>
<td>22.0</td>
<td>22.9</td>
</tr>
<tr>
<td>(A_9)</td>
<td>22.0</td>
<td>22.0</td>
<td>22.0</td>
<td>19.9</td>
<td>22.0</td>
</tr>
<tr>
<td>(A_{10})</td>
<td>1.80</td>
<td>1.62</td>
<td>1.62</td>
<td>1.80</td>
<td>1.62</td>
</tr>
<tr>
<td>Best (lb)</td>
<td>5531.98</td>
<td>5490.74</td>
<td>5490.74</td>
<td>5633.44</td>
<td>5490.74</td>
</tr>
<tr>
<td>Average (lb)</td>
<td>N/A</td>
<td>5493.49</td>
<td>5502.62</td>
<td>5838.26</td>
<td>5492.64</td>
</tr>
<tr>
<td>SD (lb)</td>
<td>N/A</td>
<td>10.46</td>
<td>20.78</td>
<td>220.39</td>
<td>2.42</td>
</tr>
<tr>
<td>Analyses</td>
<td>50000</td>
<td>5000</td>
<td>2550</td>
<td>10000</td>
<td>10000</td>
</tr>
</tbody>
</table>

![Figure 3. Convergence histories of SCA and MSCA for 10-bar truss](image-url)
It can be observed that SCA could not provide competitive results compared with the other algorithms. MSCA, AEDE and HHS find the best optimal design among other algorithms. However, the statistical results of MSCA, in terms of Average and SD are very better than those of AEDE and HHS. Moreover, the convergence rate of the MSCA is considerably better than that of the original SCA.

5.2 Example 2: 25-bar spatial truss

The 25-bar spatial truss structure, shown in Fig. 4, is one of the popular design examples in literature. The material density is 0.1 lb/in\(^3\) and the modulus of elasticity is 10\(^4\) ksi. The structure includes 25 members, which are divided into eight groups, as follows: (1) \(A_1\), (2) \(A_2-A_3\), (3) \(A_6-A_9\), (4) \(A_{10}-A_{11}\), (5) \(A_{12}-A_{13}\), (6) \(A_{14}-A_{17}\), (7) \(A_{18}-A_{21}\) and (8) \(A_{22}-A_{25}\). The allowable stress of the members is ±40 ksi and all nodes are subjected to displacement limitation of ±0.35 in.

![Figure 4. 25-bar spatial truss](image)

The design variables will be selected from the set: \([0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4]\) (in\(^2\)). The loads applied to the truss are given in Table 2.

<table>
<thead>
<tr>
<th>Node</th>
<th>(F_x) (kips)</th>
<th>(F_y) (kips)</th>
<th>(F_z) (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>-10.0</td>
<td>-10.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>-10.0</td>
<td>-10.0</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The optimization results of this study, by considering 50 particles and 100 iterations, are compared with those of HHS [21], AEDE [18] and ECBO [22] in Table 3. In addition, Fig. 5 compares the convergence curves of SCA and MSCA.
MODIFIED SINE-COSINE ALGORITHM FOR SIZING OPTIMIZATION

Table 3: Results of optimization for the 25-bar truss

<table>
<thead>
<tr>
<th>Design variables</th>
<th>HPSO</th>
<th>ECBO</th>
<th>AEDE</th>
<th>SCA</th>
<th>MSCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>$A_2$–$A_5$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$A_6$–$A_9$</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>$A_{10}$–$A_{11}$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$A_{12}$–$A_{13}$</td>
<td>2.1</td>
<td>2.1</td>
<td>2.1</td>
<td>1.4</td>
<td>2.1</td>
</tr>
<tr>
<td>$A_{14}$–$A_{17}$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>$A_{18}$–$A_{21}$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>$A_{22}$–$A_{25}$</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>Best (lb)</td>
<td>484.85</td>
<td>484.85</td>
<td>484.85</td>
<td>486.29</td>
<td>484.85</td>
</tr>
<tr>
<td>Average (lb)</td>
<td>-</td>
<td>485.89</td>
<td>485.01</td>
<td>491.17</td>
<td>484.94</td>
</tr>
<tr>
<td>SD (lb)</td>
<td>-</td>
<td>-</td>
<td>0.273</td>
<td>2.55</td>
<td>0.22</td>
</tr>
<tr>
<td>Analyses</td>
<td>25000</td>
<td>7050</td>
<td>1678</td>
<td>5000</td>
<td>5000</td>
</tr>
</tbody>
</table>

Figure 5. Convergence histories of SCA and MSCA for 10-bar truss

The numerical results indicate that, all algorithms, except SCA, converge to the best optimal design however, the Average, SD and convergence rate of MSCA are better in comparison with other algorithms. The computational effort of HPSO is significantly more than that of other algorithms. As shown in Fig. 5, the MSCA present better convergence behavior in comparison with the original SCA.

5.3 Example 3: 52-bar planar truss

Another popular benchmark truss optimization problem is the 52-bar truss shown in Fig. 6 in which $P_x$=100 kN and $P_y$=200 kN. The Young's modulus, the material density and the allowable stress are 207 GPa, 7860 kg/m$^3$ and ±180 MPa, respectively. Element groups are as: (1) $A_1$–$A_4$, (2) $A_5$–$A_{10}$, (3) $A_{11}$–$A_{13}$, (4) $A_{14}$–$A_{17}$, (5) $A_{18}$–$A_{23}$, (6) $A_{24}$–$A_{26}$, (7) $A_{27}$–$A_{30}$, (8) $A_{31}$–$A_{36}$, (9) $A_{37}$–$A_{39}$, (10) $A_{40}$–$A_{43}$, (11) $A_{44}$–$A_{49}$, and (12) $A_{50}$–$A_{52}$ which are selected from Table 4 during the optimization process. In this example, population size and maximum number of iterations are 50 and 200, respectively.
Figure 6. 52-bar truss

Table 4: Available cross-sectional areas of the AISC

<table>
<thead>
<tr>
<th>No.</th>
<th>mm$^2$</th>
<th>in$^2$</th>
<th>No.</th>
<th>mm$^2$</th>
<th>in$^2$</th>
<th>No.</th>
<th>mm$^2$</th>
<th>in$^2$</th>
<th>No.</th>
<th>mm$^2$</th>
<th>in$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71.613</td>
<td>0.111</td>
<td>17</td>
<td>1008.385</td>
<td>1.563</td>
<td>33</td>
<td>2477.414</td>
<td>3.84</td>
<td>49</td>
<td>7419.340</td>
<td>11.5</td>
</tr>
<tr>
<td>2</td>
<td>90.968</td>
<td>0.141</td>
<td>18</td>
<td>1045.159</td>
<td>1.62</td>
<td>34</td>
<td>2496.769</td>
<td>3.87</td>
<td>50</td>
<td>8709.660</td>
<td>13.5</td>
</tr>
<tr>
<td>3</td>
<td>126.451</td>
<td>0.196</td>
<td>19</td>
<td>1161.288</td>
<td>1.80</td>
<td>35</td>
<td>2503.221</td>
<td>3.88</td>
<td>51</td>
<td>8967.724</td>
<td>13.9</td>
</tr>
<tr>
<td>4</td>
<td>161.290</td>
<td>0.250</td>
<td>20</td>
<td>1283.868</td>
<td>1.99</td>
<td>36</td>
<td>2696.769</td>
<td>4.18</td>
<td>52</td>
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<td>39</td>
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<td>1.457</td>
<td>32</td>
<td>2341.931</td>
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<td>10.85</td>
<td>64</td>
<td>21612.860</td>
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</table>
Table 5 compares the optimization results of the present study and those obtained by HPSO [20], IMBA [17] and AEDE [18]. Comparison of convergence curves of SCA and MSCA is shown in Fig. 7.

Table 5: Results of optimization for the 52-bar truss

<table>
<thead>
<tr>
<th>Design variables</th>
<th>HPSO</th>
<th>IMBA</th>
<th>AEDE</th>
<th>SCA</th>
<th>MSCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{1}$–$A_{4}$</td>
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<td>4658.055</td>
<td>4658.055</td>
<td>4658.055</td>
<td>4658.055</td>
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<td>1161.288</td>
<td>1161.288</td>
<td>1161.288</td>
<td>1161.288</td>
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<td>494.193</td>
<td>494.193</td>
<td>363.225</td>
<td>494.193</td>
</tr>
<tr>
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<td>3303.219</td>
<td>3303.219</td>
<td>3303.219</td>
<td>3303.219</td>
</tr>
<tr>
<td>$A_{18}$–$A_{23}$</td>
<td>939.998</td>
<td>939.998</td>
<td>939.998</td>
<td>1045.159</td>
<td>939.998</td>
</tr>
<tr>
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<td>494.193</td>
<td>494.193</td>
<td>506.451</td>
<td>494.193</td>
</tr>
<tr>
<td>$A_{27}$–$A_{30}$</td>
<td>2238.705</td>
<td>2238.705</td>
<td>2238.705</td>
<td>2238.705</td>
<td>2238.705</td>
</tr>
<tr>
<td>$A_{31}$–$A_{36}$</td>
<td>1008.385</td>
<td>1008.385</td>
<td>1008.385</td>
<td>1008.385</td>
<td>1008.385</td>
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<tr>
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<td>494.193</td>
<td>494.193</td>
<td>641.289</td>
<td>494.193</td>
</tr>
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<td>1283.868</td>
<td>1283.868</td>
<td>1690.319</td>
<td>1283.868</td>
</tr>
<tr>
<td>$A_{44}$–$A_{49}$</td>
<td>1161.288</td>
<td>1161.288</td>
<td>1161.288</td>
<td>1045.159</td>
<td>1161.288</td>
</tr>
<tr>
<td>$A_{50}$–$A_{52}$</td>
<td>792.256</td>
<td>494.193</td>
<td>494.193</td>
<td>645.160</td>
<td>494.193</td>
</tr>
<tr>
<td>Best (kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average (kg)</td>
<td>-</td>
<td>1903.076</td>
<td>1906.735</td>
<td>1958.564</td>
<td>1904.129</td>
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<td>SD (kg)</td>
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<td>9.37</td>
<td>2.67</td>
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<tr>
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<td>4750</td>
<td>3402</td>
<td>10000</td>
<td>10000</td>
</tr>
</tbody>
</table>

It can be seen that IMBA, AEDE and MSCA converge to the same best solution and the SCA is not competitive with the other algorithms. In this example, IMBA is the best algorithm in terms of Average and SD and the second best algorithm is MSCA. Furthermore, Fig. 7 reveals that the convergence rate of the MSCA is very better than that of the original SCA.
5.4 Example 4: 72-bar spatial truss

The 72-bar spatial truss is shown in Fig. 8. In this example, there are 16 groups of elements as follows: (1) A₁–A₄, (2) A₅–A₁₂, (3) A₁₃–A₁₆, (4) A₁₇–A₁₈, (5) A₁₉–A₂₂, (6) A₂₃–A₃₀, (7) A₃₁–A₃₄, (8) A₃₅–A₃₆, (9) A₃₇–A₄₀, (10) A₄₁–A₄₈, (11) A₄₉–A₅₂, (12) A₅₃–A₅₄, (13) A₅₅–A₅₈, (14) A₅₉–A₆₆, (15) A₆₇–A₇₀, (16) A₇₁–A₇₂. The modulus of elasticity and material density are \(10^4\) ksi and 0.1 lb/in\(^3\), respectively. During the optimization process the design variables are selected from the data base of Table 4. The allowable stress in elements is ±25 ksi and the allowable horizontal displacement is ±0.25 in. In addition, there are two loading conditions given in Table 6.

![Figure 8. 72-bar truss](image)

<table>
<thead>
<tr>
<th>Node</th>
<th>Loading condition 1 (kips)</th>
<th>Loading condition 2 (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(F_x)</td>
<td>(F_y)</td>
</tr>
<tr>
<td>17</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>18</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>19</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>20</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

In the optimization process the population size and maximum number of iterations are considered to be 50 and 200, respectively. The results obtained in the present study are compared with those of HPSO [20], IMBA [17] and AEDE [18] in Table 8. Furthermore, convergence curves of SCA and MSCA are compared in Fig. 9.
Table 8: Results of optimization for the 72-bar truss

<table>
<thead>
<tr>
<th>Design variables</th>
<th>HPSO</th>
<th>IMBA</th>
<th>AEDE</th>
<th>SCA</th>
<th>MSCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1-4</td>
<td>4.97</td>
<td>1.990</td>
<td>1.990</td>
<td>3.130</td>
<td>1.990</td>
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<tr>
<td>A_5-A_12</td>
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<td>0.442</td>
<td>0.563</td>
<td>0.563</td>
<td>0.563</td>
</tr>
<tr>
<td>A_13-A_16</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
</tr>
<tr>
<td>A_17-A_18</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
</tr>
<tr>
<td>A_19-A_22</td>
<td>2.88</td>
<td>1.228</td>
<td>1.228</td>
<td>1.266</td>
<td>1.228</td>
</tr>
<tr>
<td>A_23-A_30</td>
<td>1.457</td>
<td>0.563</td>
<td>0.442</td>
<td>0.442</td>
<td>0.442</td>
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<tr>
<td>A_31-A_34</td>
<td>0.141</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
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<tr>
<td>A_35-A_36</td>
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<td>0.111</td>
<td>0.111</td>
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<tr>
<td>A_37-A_40</td>
<td>1.563</td>
<td>0.563</td>
<td>0.563</td>
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<td>0.563</td>
<td>0.442</td>
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<td>0.111</td>
<td>0.111</td>
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<td>A_53-A_54</td>
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<td>0.111</td>
<td>0.111</td>
<td>0.111</td>
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<tr>
<td>A_55-A_58</td>
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<td>0.196</td>
<td>0.196</td>
<td>0.196</td>
<td>0.196</td>
</tr>
<tr>
<td>A_59-A_66</td>
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<td>0.563</td>
<td>0.563</td>
<td>0.563</td>
<td>0.563</td>
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<tr>
<td>A_67-A_70</td>
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<td>0.391</td>
<td>0.391</td>
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<td>0.391</td>
</tr>
<tr>
<td>A_71-A_72</td>
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<td>0.563</td>
<td>0.563</td>
<td>0.442</td>
<td>0.563</td>
</tr>
<tr>
<td>Best (lb)</td>
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<td>389.33</td>
<td>389.33</td>
<td>402.96</td>
<td>389.33</td>
</tr>
<tr>
<td>Average (lb)</td>
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<td>390.91</td>
<td>412.62</td>
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<tr>
<td>SD (lb)</td>
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<td>50000</td>
<td>4160</td>
<td>10000</td>
<td>10000</td>
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</table>

These results reveal that, MSCA is competitive in comparison with other algorithms of literature. The statistical results of IMBA are slightly better than those of MSCA however at very high computational effort. In addition, it is demonstrated that the performance of MSCA is better than that of SCA.

![Convergence histories of SCA and MSCA for 72-bar truss](image)

Figure 9. Convergence histories of SCA and MSCA for 72-bar truss

5.5 Example 5: 200-bar planar truss

The 200-bar truss shown in Fig. 10 is one of the challenging truss optimization problems.
The material density, modulus of elasticity and stress limitations of the members are 0.283 lb/in$^3$, 30 Msi, and ±10 ksi, respectively. There are three loading conditions: (1) 1 kip acting in the positive x direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, and 71; (2) 10 kips acting in the negative y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71, 72, 73, 74, and 75; and (3) loading conditions 1 and 2 acting together. The variables are selected from the following database $S = \{0.100, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180, 23.680, 28.080, 33.700 \}$ in$^2$.

In this example, 60 particles at 200 iterations are employed during the optimization process. The results obtained in the present study are compared with those of improved genetic algorithm (IGA) [23], elitist self-adaptive step-size search (ESASS) [24] and AEDE [18] in Table 9. Furthermore, convergence curves of SCA and MSCA are compared in Fig. 11.
Table 9: Optimization results of the 200-bar truss

<table>
<thead>
<tr>
<th>Design variables</th>
<th>Members in the group</th>
<th>IGA</th>
<th>DE</th>
<th>AEDE</th>
<th>SCA</th>
<th>MSCA</th>
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<td>0.954</td>
<td>0.954</td>
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<td>0.1</td>
<td>1.081</td>
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<td>1.764</td>
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<td>8.525</td>
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<td>13.33</td>
<td>17.17</td>
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<tr>
<td>Best (lb)</td>
<td></td>
<td>28544</td>
<td>28075.</td>
<td>27858.</td>
<td>29611.</td>
<td>27693.6</td>
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<tr>
<td>Average (lb)</td>
<td></td>
<td>.01</td>
<td>49</td>
<td>50</td>
<td>51</td>
<td>8</td>
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<td>SD (lb)</td>
<td></td>
<td>-</td>
<td>-</td>
<td>28425.</td>
<td>31820.</td>
<td>28358.4</td>
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<td>Analyses</td>
<td></td>
<td>51360</td>
<td>-</td>
<td>11644</td>
<td>12000</td>
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The optimization results demonstrate that the proposed MSCA outperforms all other algorithms in terms of Best, Average and SD of the obtained optimal weights. Moreover, it can be observed from Fig. 11 that the convergence rate of MSCA is very better than that of the SCA.

6. CONCLUDING REMARKS

The present study focuses on a newly developed sine cosine algorithm (SCA) and proposes a modified SCA (MSCA) as the original version of this metaheuristic seriously suffers from the slow convergence rate when dealing with the discrete truss optimization problems. The proposed MSCA integrates two computational strategies during its search process. In the first strategy, named as Regeneration, a kind of elitism is utilized by substituting a number of worst solutions of the current population with some variants of the global best solution. In the second strategy, named as Mutation, a mutation operation is performed to increase the probability of finding the global optimum or near global optima.

In order to illustrate the efficiency of the MSCA, a sort of well-known discrete benchmark truss optimization problems, including 10-, 25-, 52-, 72- and 200-bar trusses, are presented and the results of MSCA are compared with those of HPSO, HHS, AEDE, ECBO, IMBA, IGA, ESASS and SCA. The numerical results demonstrate that the original SCA is not competitive with the mentioned algorithms and consequently cannot converge to appropriate solutions. In contrast, in the most cases, the proposed MSCA outperforms other algorithms and presents an appropriate performance.

REFERENCES


